Programming I

The Binary System

Dimitrios Michail

Dept. of Informatics and Telematics
Harokopio University of Athens

Decimal Numbers

A number in decimal numeral system, like 7392, represents a quantity which is equal with 7 thousands plus 3 hundreds plus 9 decades plus 2. Thousands, hundreds, etc are powers of 10 which depend on the position of the coefficient.

$$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10 + 2 \times 10^0$$

Instead of spelling out the whole expression, we simply write the coefficients and deduce the necessary power of 10 from its position.

Decimal Numbers

In general a number with a decimal point is represented by a series of coefficients such as:

$$\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0.\alpha_{-1}\alpha_{-2}\alpha_{-3}$$

every coefficient α_j is one out of 10 digits $(0, 1, 2, \dots, 9)$ and the index j denotes the position, and thus the power of 10 with which the coefficient needs to be multiplied:

$$10^5\alpha_5 + 10^4\alpha_4 + 10^3\alpha_3 + 10^2\alpha_2 + 10^1\alpha_1 + 10^0\alpha_0 + \\ 10^{-1}\alpha_{-1} + 10^{-2}\alpha_{-2} + 10^{-3}\alpha_{-3}$$

Base

We say that the decimal numeral system has **base** 10, because it uses 10 digits and the coefficients are multiplied by powers of 10.

Binary System

The binary system has a base of 2 and thus:

- uses 2 digits: 0 and 1
- coefficients are multiplied by powers of 2

Binary System

Example

$$11010 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 26$$

$$11010.11 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} = 26.75$$

System with base r

A number written in a numeral system with base r has coefficients which are multiplied by powers of r:

$$\alpha_n \cdot r^n + \alpha_{n-1} \cdot r^{n-1} + \ldots + \alpha_2 \cdot r^2 + \alpha_1 \cdot r^1 + \alpha_0 + \\ \alpha_{-1} \cdot r^{-1} + \alpha_{-2} \cdot r^{-2} + \ldots + \alpha_{-m} \cdot r^{-m}$$

the values of the coefficients α_j are between 0 and $r - 1$.

Octal System

For $r = 8$ we have the octal system which uses digits $0, \ldots, 7$ to represent numbers.

In order to distinguish numbers, we sometimes put the coefficients in parentheses and the base as a subscript:

$$(7302)_8 = 7 \cdot 8^3 + 3 \cdot 8^2 + 0 \cdot 8^1 + 2 \cdot 8^0 = (3378)_{10}$$

Hexadecimal System

For $r = 16$ we have the hexadecimal numeral system which uses the following digits:

- $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$

Digit A means 10, B means 11, etc. until F which means 15 in decimal.

$$(B65F)_{16} = 11 \cdot 16^3 + 6 \cdot 16^2 + 5 \cdot 16 + 15 = (46687)_{10}$$

Numbers in Different Bases

Decimal	Binary	Octal	Hexadecimal
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Material

- Chapter 1:
  Digital Design, Mano Morris and Ciletti Michael. Pearson. 6th edition. 2017.
- Wikipedia article:
  http://en.wikipedia.org/wiki/Binary_numeral_system